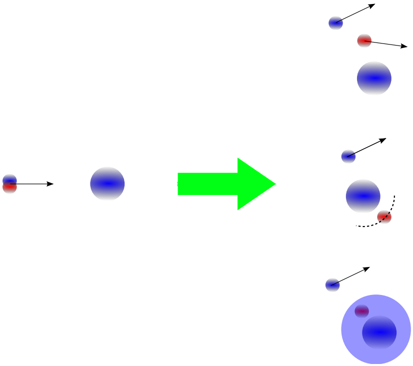
Deuteron–induced reactions

Grégory Potel Aguilar (NSCL, LLNL)
Filomena Nunes (NSCL)
Ian Thompson (LLNL)

INT, March 2015

Introduction

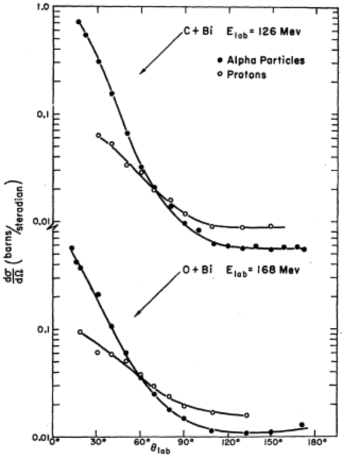
We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:



- Direct neutron transfer: should be compatible with existing theories.
- Elastic deuteron breakup: **"transfer" to continuum states**.
- Neutron capture and compound nucleus formation: **absorption above and below neutron emission threshold**.
- Important application in surrogate reactions: **obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation** (see J. Escher's talk).

Historical background

breakup-fusion reactions



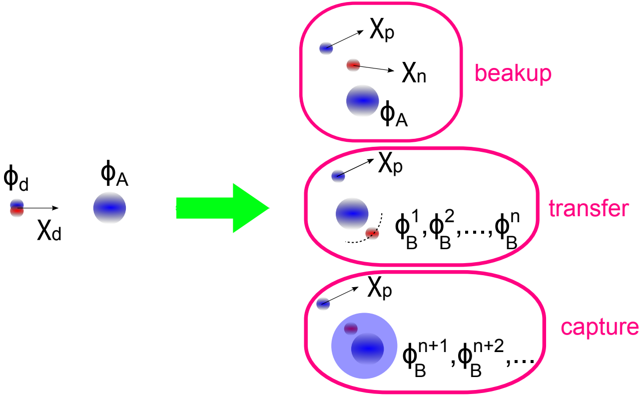
Britt and Quinton, Phys. Rev. **124** (1961) 877

protons and α yields bombarding ^{209}Bi with ^{12}C and ^{16}O

- Kerman and McVoy, Ann. Phys. **122** (1979)197
- Austern and Vincent, Phys. Rev. C**23** (1981) 1847
- Udagawa and Tamura, Phys. Rev. C**24**(1981) 1348
- Last paper: Mastroleo, Udagawa, Mustafa Phys. Rev. C**42** (1990) 683
- Controversy between Udagawa and Austern formalism left somehow unresolved.

Inclusive (d, p) reaction

let's concentrate in the reaction A+d→ B(=A+n)+p



we are interested in the inclusive cross section, *i.e.*, we will sum over all final states ϕ_B^c.

Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final ϕ_B^c

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\left|\left\langle \chi_p \phi_B^c | V | \Psi^{(+)} \right\rangle\right|^2 .$$

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d}\rho(E_p)$$
$$\times \sum_c \langle \chi_d \phi_d \phi_A | \, V \, | \chi_p \phi_B^c \rangle \, \delta(E - E_p - E_B^c) \, \langle \phi_B^c \chi_p | \, V \, | \phi_A \chi_d \phi_d \rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,
$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.

Sum over final states

the imaginary part of the Green's function G is an operator representation of the δ–function,

$$\pi\delta(E - E_p - E_B^c) = \lim_{\epsilon\to 0} \Im \sum_c \frac{|\phi_B^c\rangle \langle\phi_B^c|}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d}\rho(E_p)\Im \langle \chi_d\phi_d\phi_A| V |\chi_p\rangle G \langle\chi_p| V |\phi_A\chi_d\phi_d\rangle$$

- We got rid of the (infinite) sum over final states,
- but G is an extremely complex object!
- We still need to deal with that.

Optical reduction of G

If the interaction V do not act on ϕ_A

$$\langle \chi_d \phi_d \phi_A | \, V \, | \chi_p \rangle \, G \, \langle \chi_p | \, V \, | \phi_A \chi_d \phi_d \rangle = \langle \chi_d \phi_d | \, V \, | \chi_p \rangle \, G_{opt} \, \langle \chi_p | \, V \, | \chi_d \phi_d \rangle \, ,$$

where G_{opt} is the optical reduction of G

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon} ,$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus G_{opt} are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations (previous talks by W. Dickhoff, C. Barbieri, P. Navratil, G. Hagen, J. Rotureau, J. Holt...)

Capture and elastic breakup cross sections

the imaginary part of G_{opt} splits in two terms

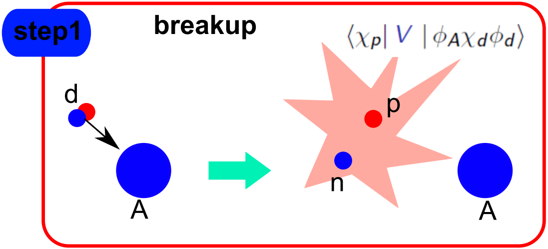
$$\Im G_{opt} = \overbrace{-\pi \sum_{k_n} |\chi_n\rangle \delta\left(E - E_p - \frac{k_n^2}{2m_n}\right)\langle\chi_n|}^{\text{elastic breakup}} + \overbrace{G_{opt}{}^{\dagger} W_{An}\, G_{opt}}^{\text{neutron capture}},$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt}\langle\chi_p|\, V\, |\chi_d\phi_d\rangle$

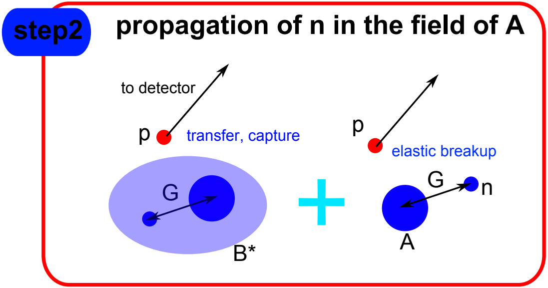
cross sections for neutron capture and elastic breakup

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{capture} = -\frac{2}{\hbar v_d}\rho(E_p)\langle\psi_n|\, W_{An}\, |\psi_n\rangle\,,$$

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{breakup} = -\frac{2}{\hbar v_d}\rho(E_p)\rho(E_n)\left|\langle\chi_n\chi_p|\, V\, |\chi_d\phi_d\rangle\right|^2,$$



step1
breakup
$\langle \chi_p | \, V \, | \phi_A \chi_d \phi_d \rangle$
d
A
p
n
A



step2
propagation of n in the field of A
to detector
p
transfer, capture
G
B*
p
elastic breakup
G
n
A

Austern (post)–Udagawa (prior) controversy

The interaction V can be taken either in the *prior* or the *post* representation,

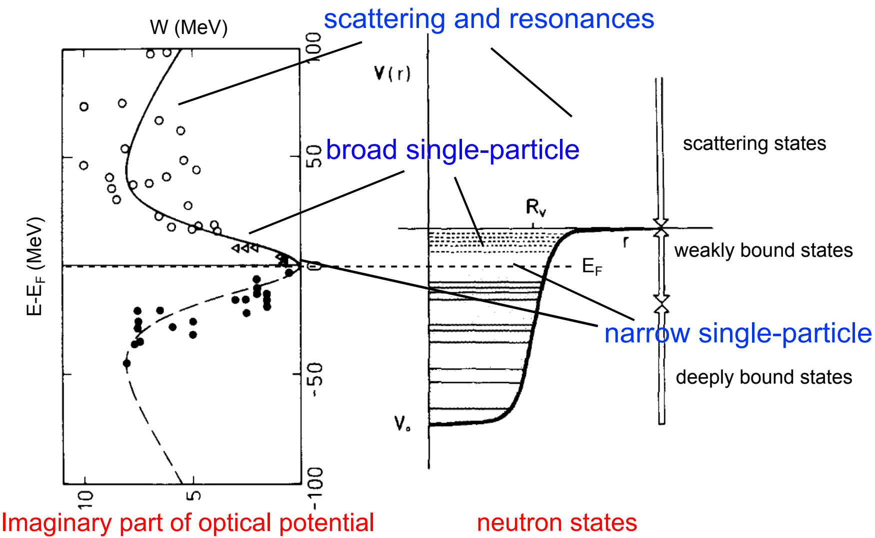
- Austern (post)$\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$

in the prior representation, V can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{post} = -\frac{2}{\hbar v_d}\rho(E_p)\left[\Im\left\langle \psi_n^{prior} | W_{An} | \psi_n^{prior} \right\rangle \right. \\ \left. + 2\Re\left\langle \psi_n^{NON} | W_{An} | \psi_n^{prior} \right\rangle + \left\langle \psi_n^{NON} | W_{An} | \psi_n^{NON} \right\rangle \right],$$

where $\psi_n^{NON} = \langle \chi_p | \ \chi_d \phi_d \rangle$.

Neutron states in nuclei



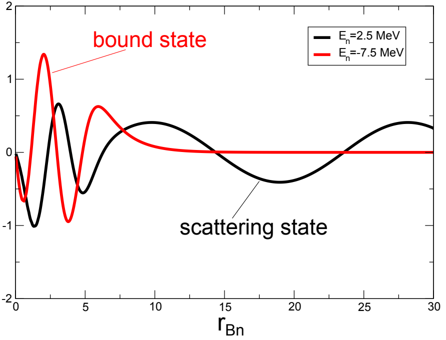
Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1

neutron wavefunctions

the neutron wavefunctions

$$|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$$

can be computed for any neutron energy



transfer to resonant and non-resonant continuum well described

these wavefunctions are not eigenfunctions of the Hamiltonian
$H_{An} = T_n + \Re(U_{An})$

neutron transfer limit (isolated–resonance, first–order approximation)

Let's consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy E such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle\, \chi_d \phi_d |\, V \,|\chi_p\rangle$$
$$\times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle\chi_p|\, V \,|\, \chi_d \phi_d\rangle\,,$$

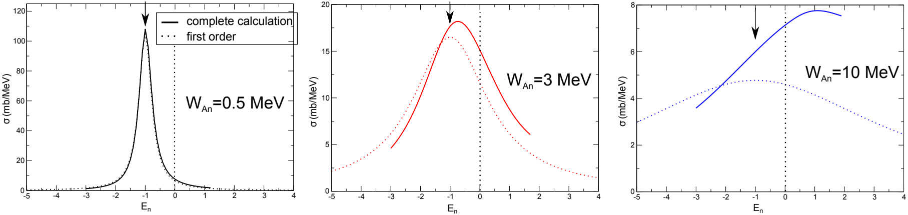
we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\, \langle\chi_p \phi_n|\, V \,|\, \chi_d \phi_d\rangle\, |^2 \delta(E - E_p - E_n)$$

Validity of first order approximation

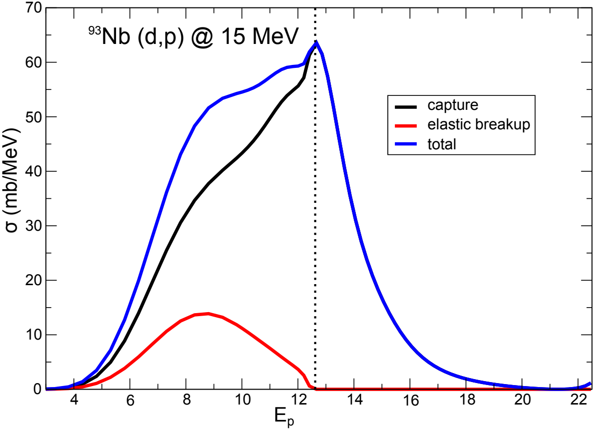
For W_{An} small, we can apply first order perturbation theory,

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}(E,\Omega)\right]^{capture} \approx \frac{1}{\pi}\frac{\langle\phi_n|W_{An}|\phi_n\rangle}{(E_n-E)^2+\langle\phi_n|W_{An}|\phi_n\rangle^2}\left.\frac{d\sigma_n}{d\Omega}(\Omega)\right]^{transfer}$$



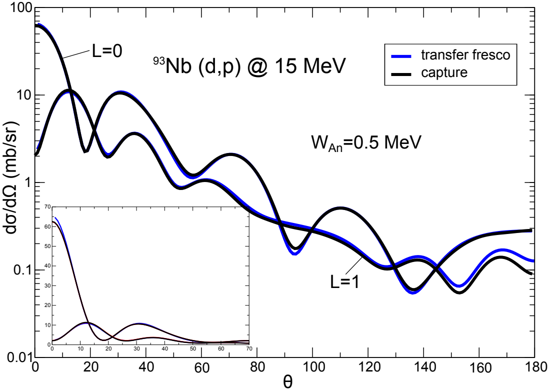
we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 0.5$ MeV and $W_{An} = 0.5$ MeV

Observables: elastic breakup and capture cross sections



elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the U_{An} interaction (Koning and Delaroche, Nucl. Phys. A **713** (2003) 231).

Observables: angular differential cross sections (neutron bound states)

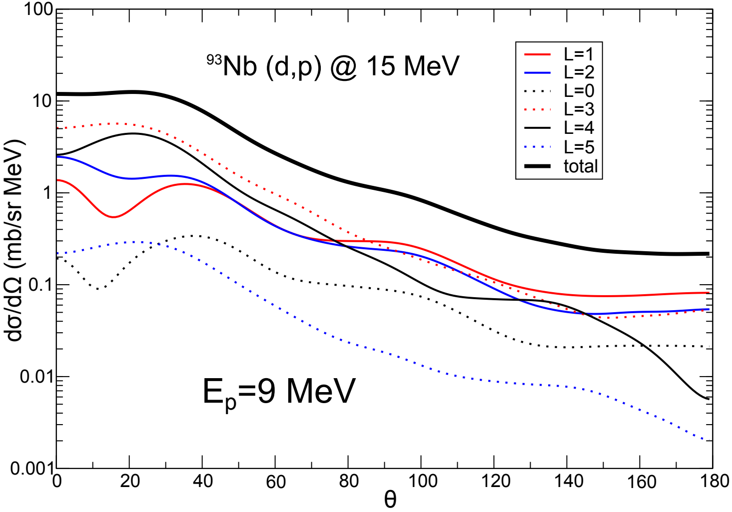


- capture at resonant energies compared with
- direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

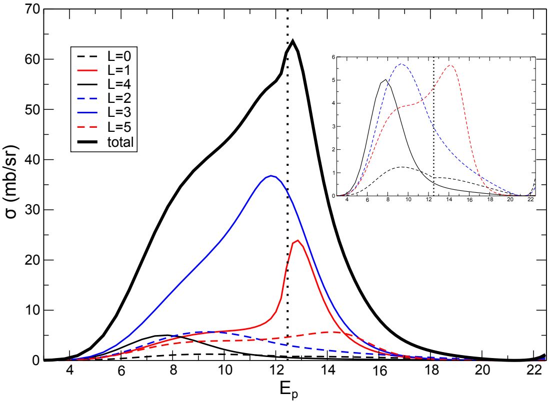
double proton differential cross section

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y^{l_p}_{-m}(\theta_p) \right|^2 W(r_{An})\ dr_{Bn}.$$

Observables: angular differential cross sections (above neutron–emission threshold)



Observables: compound nucleus spin and parity

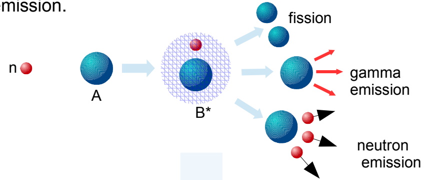


spin distribution of compound nucleus

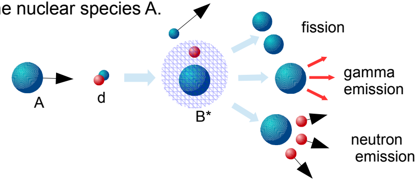
$$\frac{d\sigma_l}{dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\sum_{l_p,m}\int \left|\varphi_{lml_p}(r_{Bn};k_p)\right|^2 W(r_{An})\, dr_{Bn}.$$

Surrogate for neutron capture

* Desired reaction: neutron induced fission, gamma emission and neutron emission.

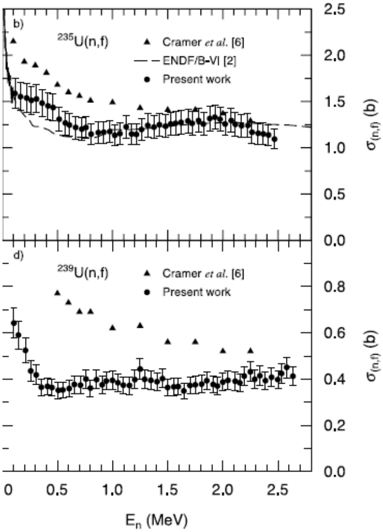


* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.



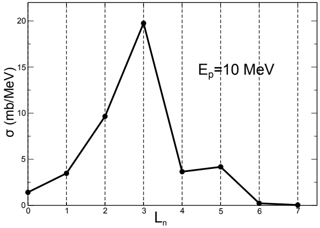
* **A theoretical reaction formalism that describes the production of all open channels B* is needed.**

Surrogate reactions



Younes and Britt, PRC **68**(2003)034610

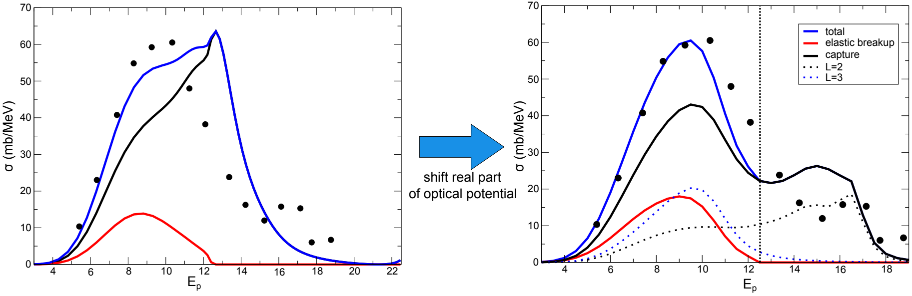
- Weisskopf–Ewing approximation: $P(d,nx) = \sigma(E)G(E,x)$
- inaccurate for $x = \gamma$ and for $x = f$ in the low–energy regime
- can be replaced by $P(d,nx) = \sum_{J,\pi} \sigma(E,J,\pi)G(E,J,\pi,x)$ if $\sigma(E,J,\pi)$ can be predicted.



(see J. Escher talk)

Preliminary comparison with experiment

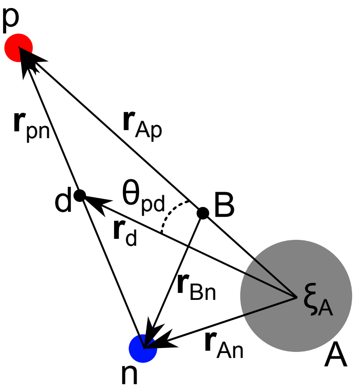
We show very preliminary results for the ^{93}Nb(d,p) reaction with a 15 MeV deuteron beam (Mastroleo *et al.*, Phys. Rev. C **42** (1990) 683)



- we have used the Koning–Delaroche optical potential
- the real part of the optical potential has been shifted to reproduce the position of the $L = 3$ resonance
- the experimental results seem to be sensitive to the position and strength of a modest number of resonances

Summary, conclusions and some prospectives

- Reaction formalism for inclusive deuteron–induced reaction.
- final neutron states from Fermi energy $\rightarrow$ to scattering states
- 2–step reaction mechanism $\rightarrow$ breakup+absorption
- probe of nuclear structure over a wide energy range
- need for optical potentials
- useful for surrogate reactions
- transfer to individual resonances?
- extend for (p, d) reactions (hole states)?



From H to H_{3B}

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$